

Chapter 3

Modeling

Topics to be covered include:

- How to select the appropriate model complexity
- How to build models for a given plant
- How to describe model errors.
- How to linearize nonlinear models

It also provides a brief introduction to certain commonly used models, including

- State space models
- High order differential and high order difference equation models

The Raison *d'être* for Models

The basic idea of feedback is tremendously compelling. Recall the mould level control problem from Chapter 2. Actually, there are only three ways that a controller could manipulate the valve: open, close or leave it as it is. Nevertheless, we have seen already that the precise way this is done involves subtle trade-offs between conflicting objectives, such as speed of response and sensitivity to measurement noise.

The power of a mathematical model lies in the fact that it can be simulated in hypothetical situations, be subject to states that would be dangerous in reality, and it can be used as a basis for synthesizing controllers.

Model Complexity

In building a model, it is important to bear in mind that all real processes are complex and hence any attempt to build an exact description of the plant is usually an impossible goal. Fortunately, feedback is usually very forgiving and hence, in the context of control system design, one can usually get away with rather simple models, provided they capture the essential features of the problem.

We introduce several terms:

- **Nominal model**. This is an approximate description of the plant used for control system design.
- **Calibration model**. This is a more comprehensive description of the plant. It includes other features not used for control system design but which have a direct bearing on the achieved performance.
- **Model error**. This is the difference between the nominal model and the calibration model. Details of this error may be unknown but various bounds may be available for it.

Building Models

A first possible approach to building a plant model is to postulate a specific model structure and to use what is known as a *black box* approach to modeling. In this approach one varies, either by trial and error or by an algorithm, the model parameters until the dynamic behavior of model and plant match sufficiently well.

An alternative approach for dealing with the modeling problem is to use physical laws (such as conservation of mass, energy and momentum) to construct the model. In this approach one uses the fact that, in any real system, there are *basic phenomenological laws* which determine the relationships between all the signals in the system.

In practice, it is common to combine both black box and phenomenological ideas to building a model.

Control relevant models are often quite simple compared to the true process and usually combine physical reasoning with experimental data.

State Space Models

For continuous time systems

$$\frac{dx}{dt} = f(x(t), u(t), t)$$
$$y(t) = g(x(t), u(t), t)$$

For discrete time systems

$$x[k+1] = f_d(x[k], u[k], k)$$
$$y[k] = g_d(x[k], u[k], k)$$

Linear State Space Models

$$\frac{dx(t)}{dt} = \mathbf{A}x(t) + \mathbf{B}u(t)$$
$$y(t) = \mathbf{C}x(t) + \mathbf{D}u(t)$$

Example 3.3

Consider the simple electrical network shown in *Figure 3.1*. Assume we want to model the voltage $v(t)$

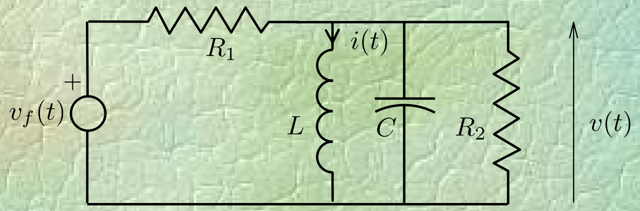


Figure 3.1: *Electrical network. State space model.*

On applying fundamental network laws we obtain the following equations:

$$v(t) = L\frac{di(t)}{dt}$$

$$\frac{v_f(t) - v(t)}{R_1} = i(t) + C\frac{dv(t)}{dt} + \frac{v(t)}{R_2}$$

These equations can be rearranged as follows:

$$\frac{di(t)}{dt} = \frac{1}{L}v(t)$$
$$\frac{dv(t)}{dt} = -\frac{1}{C}i(t) - \left(\frac{1}{R_1C} + \frac{1}{R_2C}\right)v(t) + \frac{1}{R_1C}v_f(t)$$

We have a linear state space model with

$$\mathbf{A} = \begin{bmatrix} 0 & \frac{1}{L} \\ -\frac{1}{C} & -\left(\frac{1}{R_1C} + \frac{1}{R_2C}\right) \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} 0 \\ \frac{1}{R_1C} \end{bmatrix}; \quad \mathbf{C} = \begin{bmatrix} 0 & 1 \end{bmatrix}; \quad \mathbf{D} = \mathbf{0}$$

Example 3.4

Consider a separately excited d.c. motor. Let $v_a(t)$ denote the armature voltage, $\theta(t)$ the output angle. A simplied schematic diagram of this system is shown in Figure 3.2.

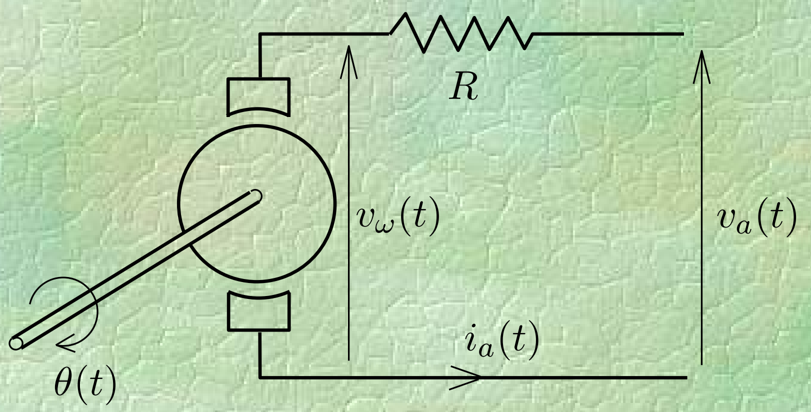
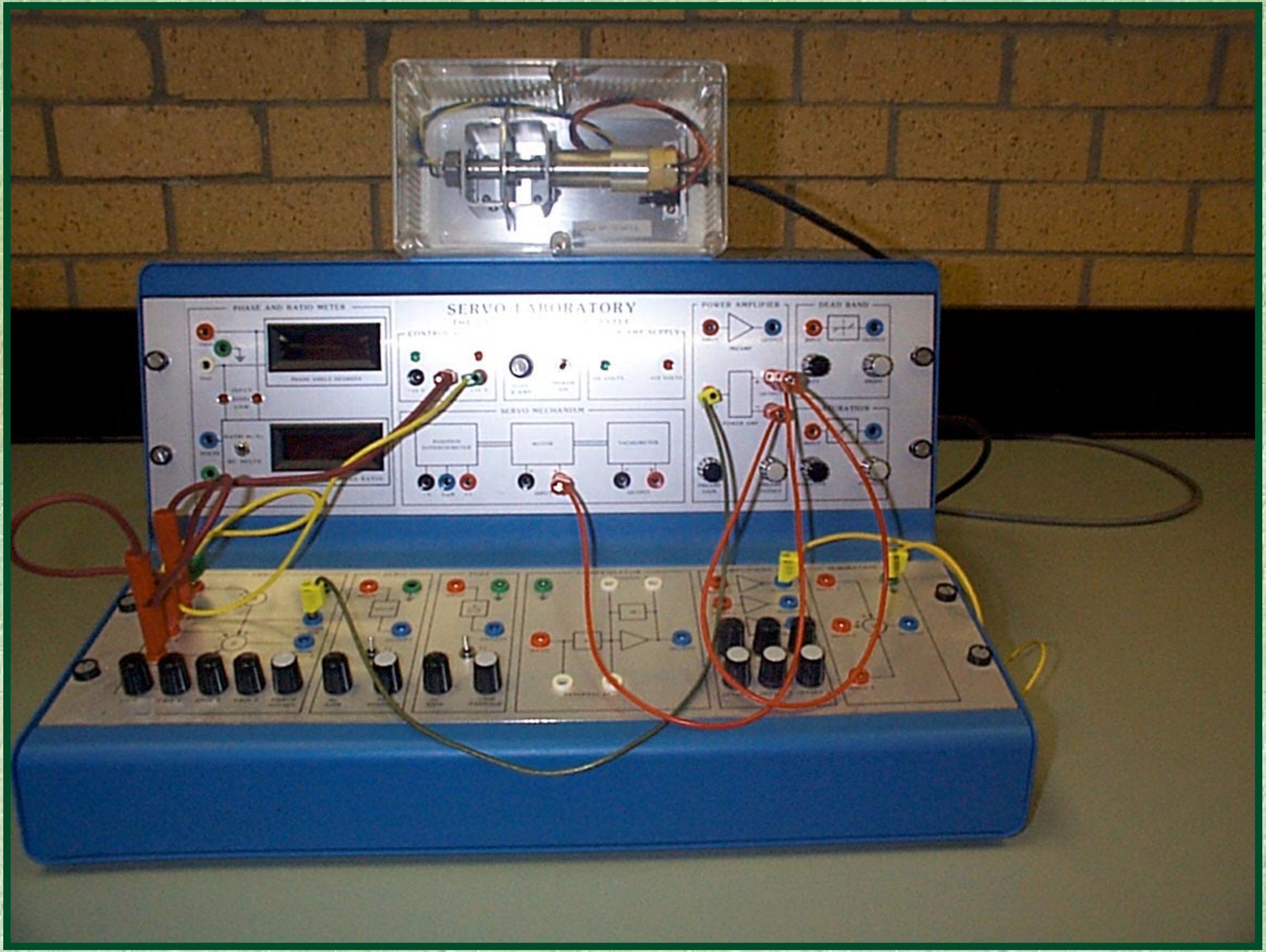
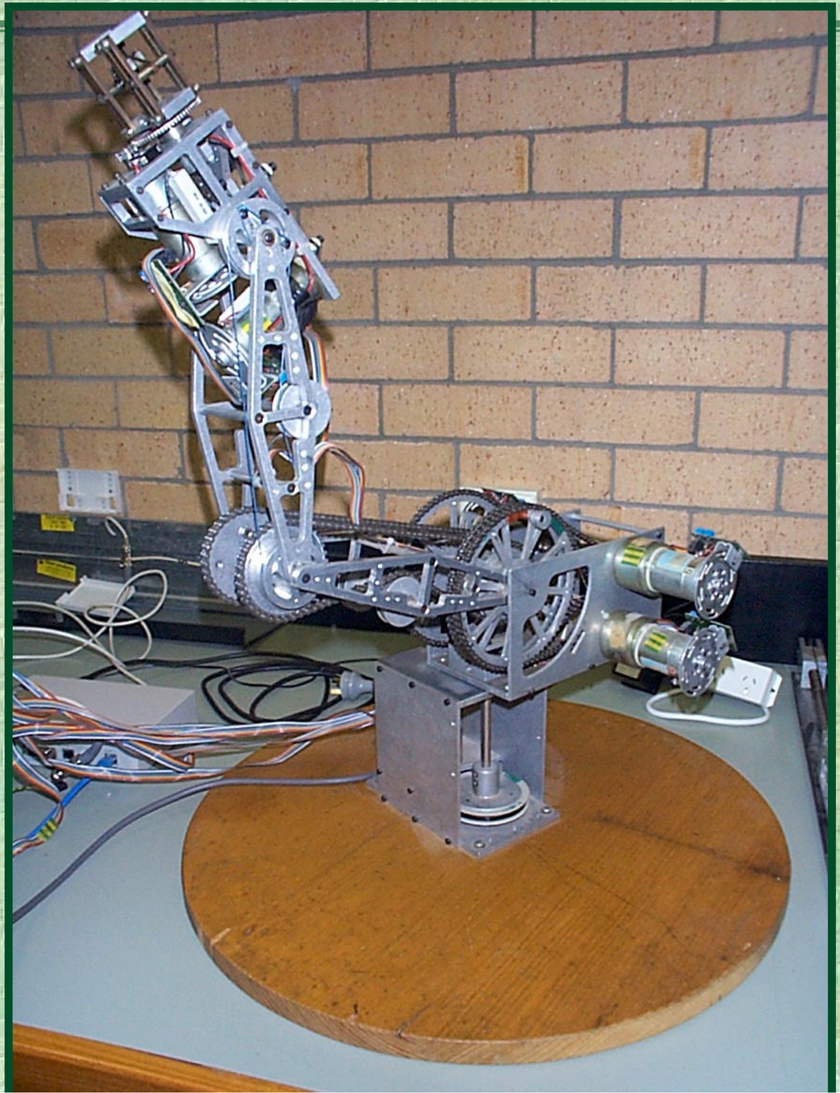


Figure 3.2: *Simplified model of a d.c. motor*

A laboratory servo kit



A demonstration robot containing several servo motors



Let

J	- be the inertia of the shaft
$\tau_e(t)$	- the electrical torque
$i_a(t)$	- the armature current
$k_1; k_2$	- constants
R	- the armature resistance

Application of well known principles of physics tells us that the various variables are related by:

$$J\ddot{\theta}(t) = \tau_e(t) = k_1 i_a(t)$$

$$v_\omega(t) = k_2\dot{\theta}(t)$$

$$i_a(t) = \frac{v_a(t) - k_2\dot{\theta}(t)}{R}$$

$$\frac{d}{dt}\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & \frac{-k_1 k_2}{R} \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{k_1}{R} \end{bmatrix} v_a(t)$$

Solution of Continuous Time State Space Models

A key quantity in determining solutions to state equations is the *matrix exponential* defined as

$$e^{\mathbf{A}t} = \mathbf{I} + \sum_{i=1}^{\infty} \frac{1}{i!}\mathbf{A}^i t^i$$

The explicit solution to the linear state equation is then given by

$$x(t) = e^{\mathbf{A}(t-t_o)}x_o + \int_{t_o}^{t} e^{\mathbf{A}(t-\tau)}\mathbf{B}u(\tau)d\tau$$

Modeling Errors

The so-called *additive modeling error* (AME) is defined by a transformation g_2 such that

$$y = y_o + g_\epsilon\langle u\rangle$$

A difficulty with the AME is that it is not scaled relative to the *size* of the nominal model. This is the advantage of the so-called *multiplicative modeling error* (MME), g_Δ , defined by

$$y = g_o\langle u + g_\Delta\langle u\rangle\rangle$$

Example 3.5

The output of a plant is assumed to be exactly described by

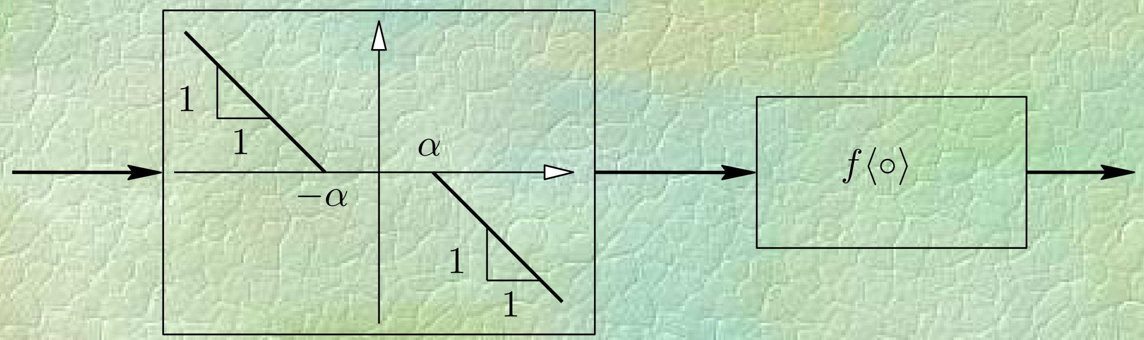
$$y = f\Big\langle sat_\alpha\langle u\rangle\Big\rangle$$

where $f\langle\circ\rangle$ *is a linear transformation and sat denotes the saturation operator, i.e.*

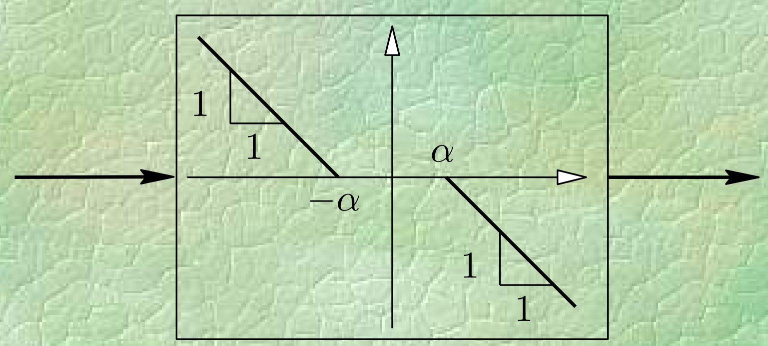
$$sat_\alpha\langle x\rangle = \begin{cases} \alpha & |x(t)| > |\alpha| \\ x & |x(t)| \leq |\alpha| \end{cases}$$

If the nominal model is chosen as $g_0\langle\circ\rangle = f\langle\circ\rangle$*, i.e. the saturation is ignored, determine the additive and the multiplicative modeling errors.*

Figure 3.3: *AME and MME due to saturation*



Additive modelling error



Multiplicative modelling error

Linearization

Although almost every real system includes nonlinear features, many systems can be reasonably described, at least within certain operating ranges, by linear models.

Thus consider

$$\dot{x}(t) = f(x(t), u(t))$$
$$y(t) = g(x(t), u(t))$$

Say that $\{x_Q(t), u_Q(t), y_Q(t); t \in \mathbb{R}\}$ is a given set of trajectories that satisfy the above equations, i.e.

$$\dot{x}_Q(t) = f(x_Q(t), u_Q(t)); \qquad x_Q(t_o) \text{ given}$$
$$y_Q(t) = g(x_Q(t), u_Q(t))$$

$$\dot{x}(t) \approx f(x_Q, u_Q) + \left.\frac{\partial f}{\partial x}\right|_{\substack{x=x_Q\\u=u_Q}} (x(t) - x_Q) + \left.\frac{\partial f}{\partial u}\right|_{\substack{x=x_Q\\u=u_Q}} (u(t) - u_Q)$$

$$y(t) \approx g(x_Q, u_Q) + \left.\frac{\partial g}{\partial x}\right|_{\substack{x=x_Q\\u=u_Q}} (x(t) - x_Q) + \left.\frac{\partial g}{\partial u}\right|_{\substack{x=x_Q\\u=u_Q}} (u(t) - u_Q)$$

$$\dot{x}(t) = \mathbf{A}x(t) + \mathbf{B}u(t) + \mathbf{E}$$
$$y(t) = \mathbf{C}x(t) + \mathbf{D}u(t) + \mathbf{F}$$

$$\mathbf{A} = \left.\frac{\partial f}{\partial x}\right|_{\substack{x=x_Q\\u=u_Q}} ; \qquad \mathbf{B} = \left.\frac{\partial f}{\partial u}\right|_{\substack{x=x_Q\\u=u_Q}}$$

$$\mathbf{C} = \left.\frac{\partial g}{\partial x}\right|_{\substack{x=x_Q\\u=u_Q}} ; \qquad \mathbf{D} = \left.\frac{\partial g}{\partial u}\right|_{\substack{x=x_Q\\u=u_Q}}$$

$$\mathbf{E} = f(x_Q, u_Q) - \left.\frac{\partial f}{\partial x}\right|_{\substack{x=x_Q\\u=u_Q}} x_Q - \left.\frac{\partial f}{\partial u}\right|_{\substack{x=x_Q\\u=u_Q}} u_Q$$

$$\mathbf{F} = g(x_Q, u_Q) - \left.\frac{\partial g}{\partial x}\right|_{\substack{x=x_Q\\u=u_Q}} x_Q - \left.\frac{\partial g}{\partial u}\right|_{\substack{x=x_Q\\u=u_Q}} u_Q$$

Example 3.6

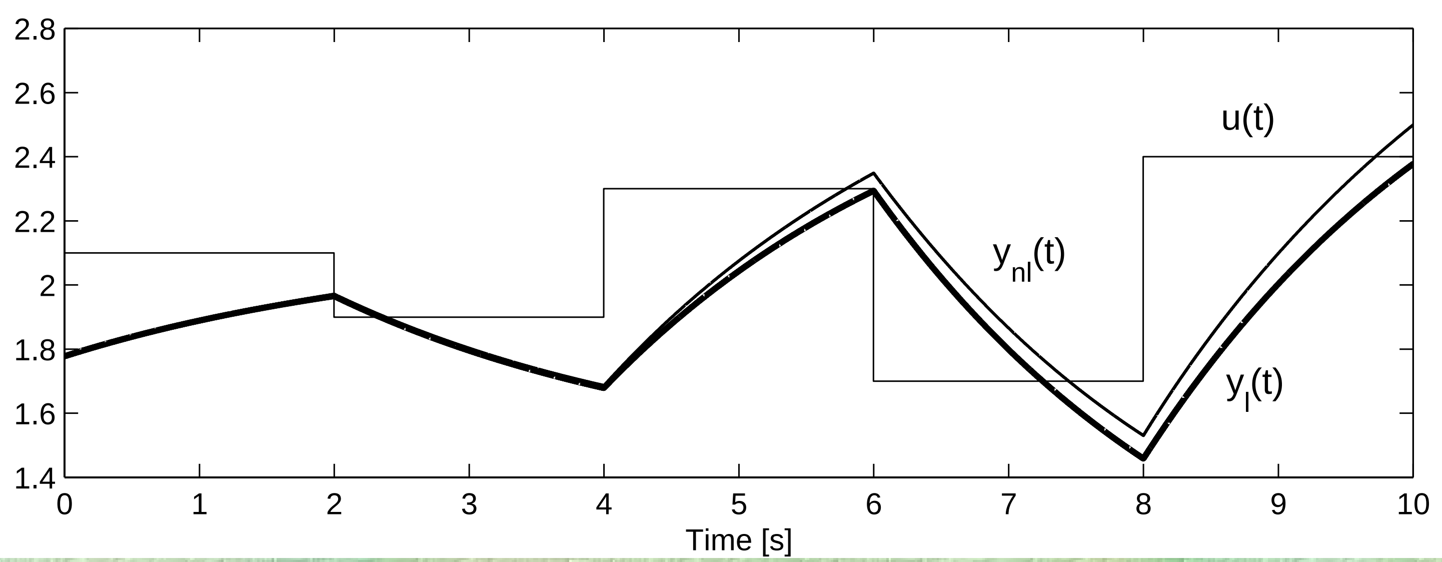
Consider a continuous time system with true model given by

$$\frac{dx(t)}{dt} = f(x(t), u(t)) = -\sqrt{x(t)} + \frac{(u(t))^2}{3}$$

Assume that the input $u(t)$ fluctuates around $u = 2$. Find an operating point with $u_Q = 2$ and a linearized model around it.

$$\frac{d\Delta x(t)}{dt} = -\frac{3}{8}\Delta x(t) + \frac{4}{3}\Delta u(t)$$

Figure 3.4: *Nonlinear system output, $y_{nl}(t)$, and linearized system output, $y_l(t)$, for a square wave input of increasing amplitude, $u(t)$.*



Example 3.7 (Inverted pendulum)

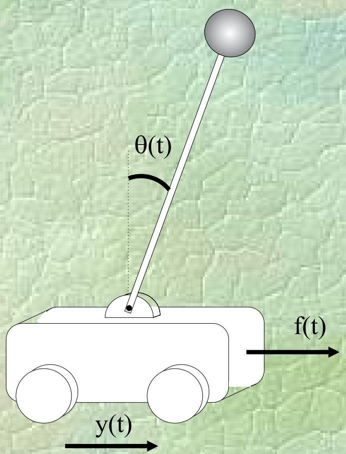


Figure 3.5: *Inverted pendulum*

In Figure 3.5, we have used the following notation:

$y(t)$	-	distance from some reference point
$\theta(t)$	-	angle of pendulum
M	-	mass of cart
m	-	mass of pendulum (assumed concentrated at tip)
ℓ	-	length of pendulum
$f(t)$	-	forces applied to pendulum

Example of an Inverted Pendulum

Application of Newtonian physics to this system leads to the following model:

$$\ddot{y} = \frac{1}{\lambda_m + \sin^2\theta(t)}\left[\frac{f(t)}{m} + \dot{\theta}^2(t)\ell\sin\theta(t) - g\cos\theta(t)\sin\theta(t)\right]$$

$$\ddot{\theta} = \frac{1}{\ell\lambda_m + \sin^2\theta(t)}\left[-\frac{f(t)}{m}\cos\theta(t) + \dot{\theta}^2(t)\ell\sin\theta(t)\cos\theta(t) + (1-\lambda_m)g\sin\theta(t)\right]$$

where $\lambda_m = (M/m)$

This is a linear state space model in which A, B and C are:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{-mg}{M} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{(M+m)g}{M\ell} & 0 \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \\ -\frac{1}{M\ell} \end{bmatrix}; \quad \mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

Further Examples

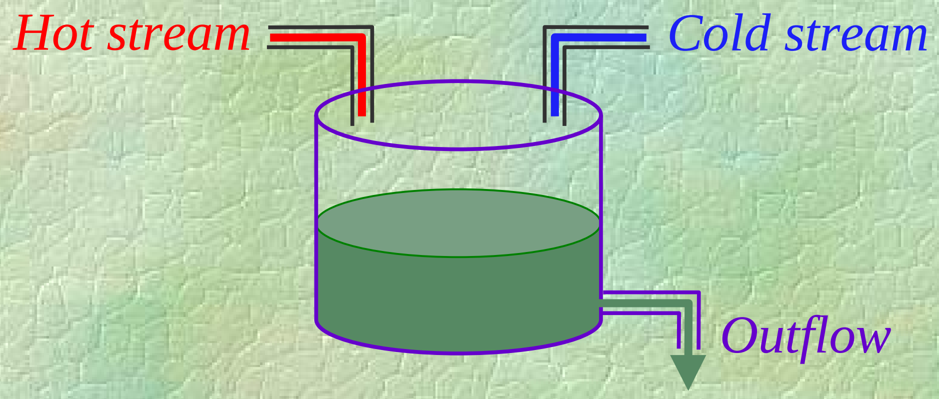
Here we present several other examples which illustrate basic modelling principles.

All of these models depend upon basic physical principles, e.g.

- conservation of mass
- conservation of momentum
- conservation of energy.

Further Example 1:Thermal Mixing

Consider a cylindrical tank into which flows hot and cold streams of fluid



Tank is assumed well stirred and well insulated.

Let	u_h	=	hot stream flow rate
	u_c	=	cold stream flows rate
	A	=	Area of tank
	h	=	level of liquid in tank
	T	=	temperature of outflow
	f_{out}	=	outflow rate
	T_h	=	temperature of hot stream
	T_c	=	temperature of cold stream

We assume we know u_h, u_c, A, T_h, T_c and want to model T and h. Also assume that all streams have a common specific heat c and density ρ.

Also, we assume

$$f_{out} = k\sqrt{h}$$

Mass Balance

Rate of Change of Volume $= u_h + u_c - f_{out}$

$$A\frac{dh}{dt} = u_h + u_c - k\sqrt{h}$$

Energy Balance

Rate of Change of Internal Energy = heat in - heat out

$$A\rho c \frac{dTh}{dt} = \rho c u_h T_h + \rho c u_c T_c - \rho c f_{out} T$$

Combining these equations we obtain

$$A\frac{dh}{dt} = u_h + u_c - k\sqrt{h}$$

$$Ah\frac{dT}{dt} + AT\frac{dh}{dt} = u_h T_h + u_c T_c - kT\sqrt{h}$$

Final Nonlinear Model

Substituting the first equation above into the second yields:

$$\frac{dh}{dt} = -\frac{k}{A}\sqrt{h} + \frac{u_h}{A} + \frac{u_c}{A}$$

$$\frac{dT}{dt} = \frac{1}{Ah}\{(T_h - T)u_h + (T_c - T)u_c\}$$

Observations

We see that the model comprises 2 *nonlinear* state space equations. (*The nonlinearities appear in the* $\sqrt{h}$ *term and in terms of the form* $\left[\frac{Tu}{h}\right]$).

We next simulate the system to get a feel for its behaviour.

Parameters

For simplicity, we assume

$$A = 1, \qquad k = 1$$

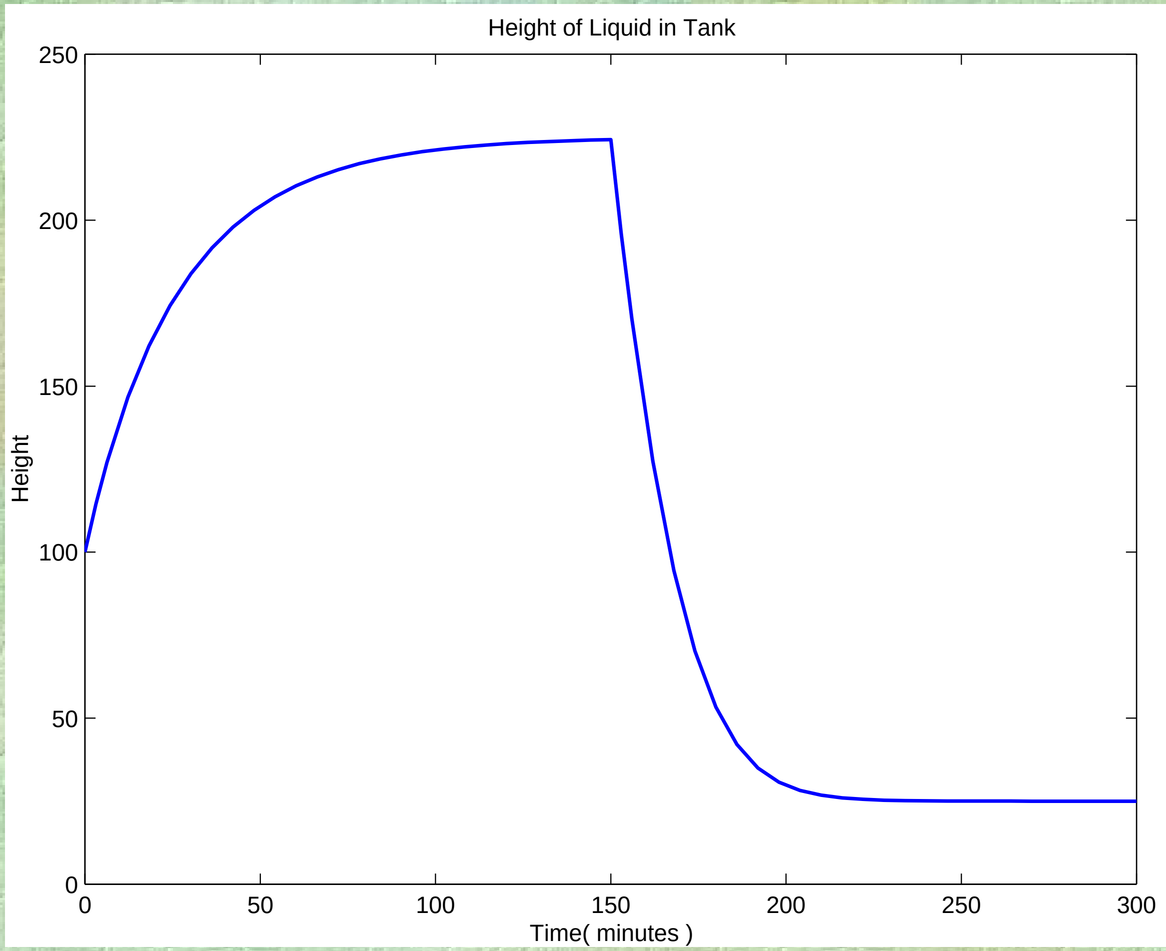
$$T_h = 50, \qquad T_c = 20$$

Simulation 1

We begin by using only the hot water flow as input.

We begin with a flow of 10, then step it up to 15 and then back to 5.

Note that the temperature stays constant at 50ºC. The height response is shown on the next slide.


Height of Liquid in Tank
250
200
150
100
50
0
Height
0
50
100
150
200
250
300
Time(minutes)

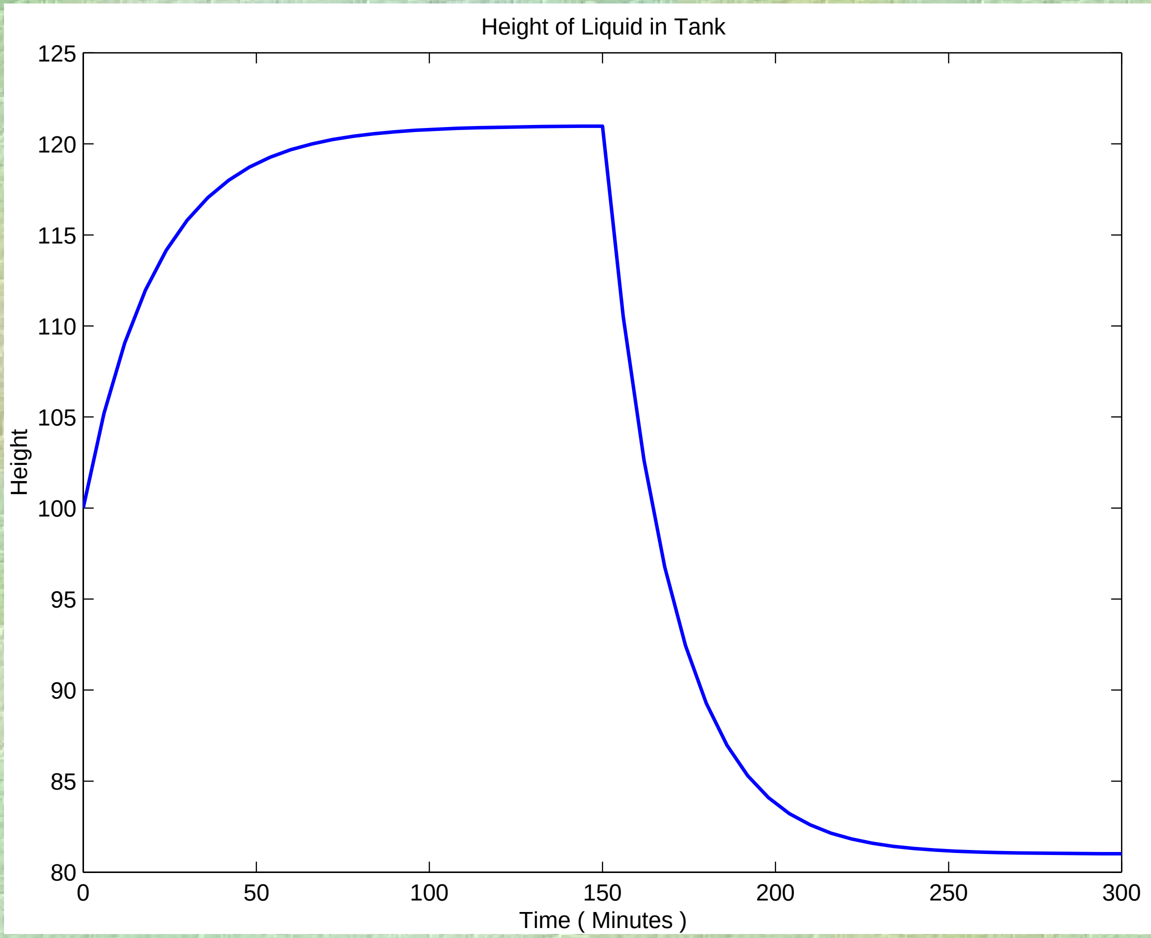
Observations

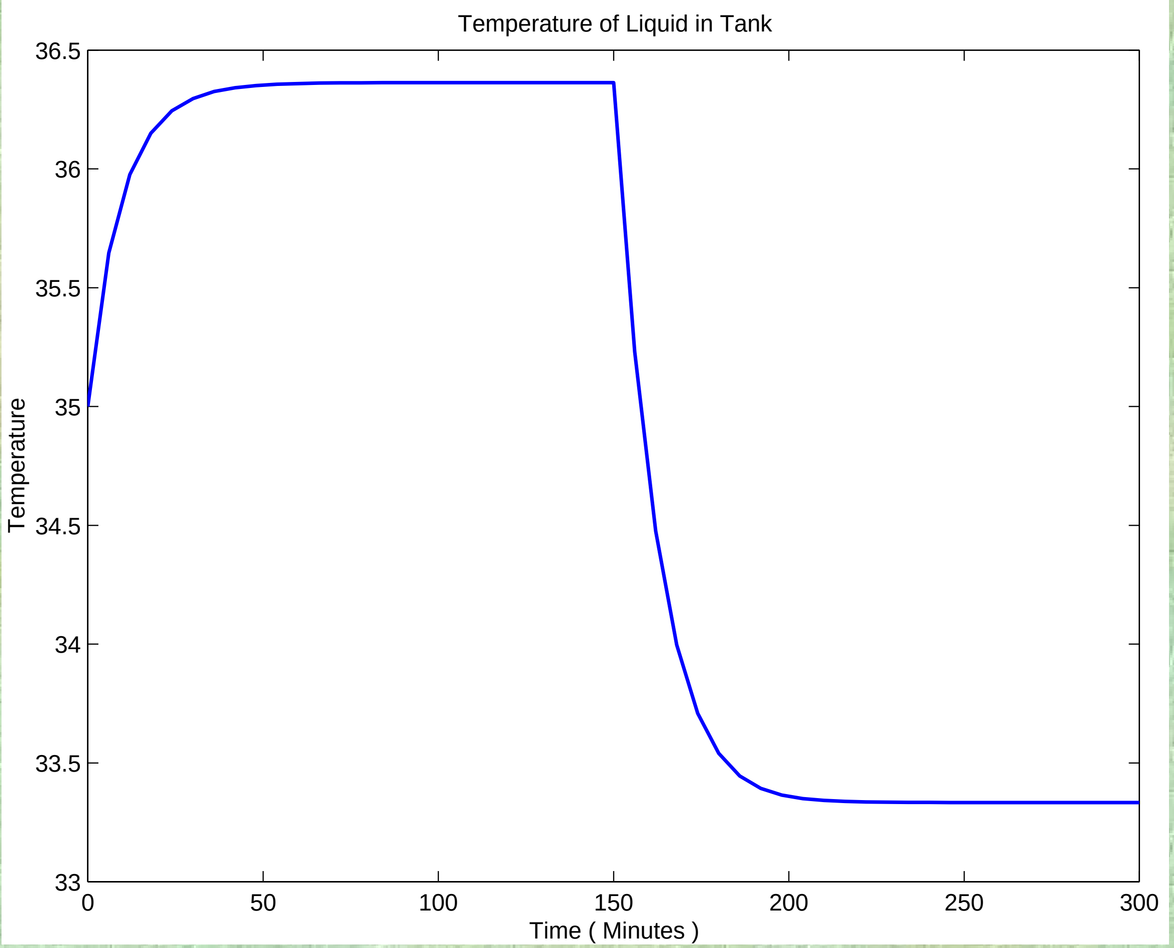
- When we increase the hot flow the level goes up (*in a roughly exponential fashion*).
- Notice the nonlinear behaviour:
  - the settling time when the flow is increased is about 150 minutes compared to 50 minutes when the flow is decreased.
  - Increasing the flow by 5 gives an increase in level of 125 but decreasing the flow by 5 gives a decrease in level of 75.

Simulation 2

We next set both flows to 5. We then step up the hot flow to 6 followed by decrease to 4.

The results are shown in the next plots.


Height of Liquid in Tank
Height
Time (Minutes)
125
120
115
110
105
100
95
90
85
80
0
50
100
150
200
250
300


Temperature of Liquid in Tank
36.5
36
35.5
35
34.5
34
33.5
33
Temperature
0
50
100
150
200
250
300
Time (Minutes)

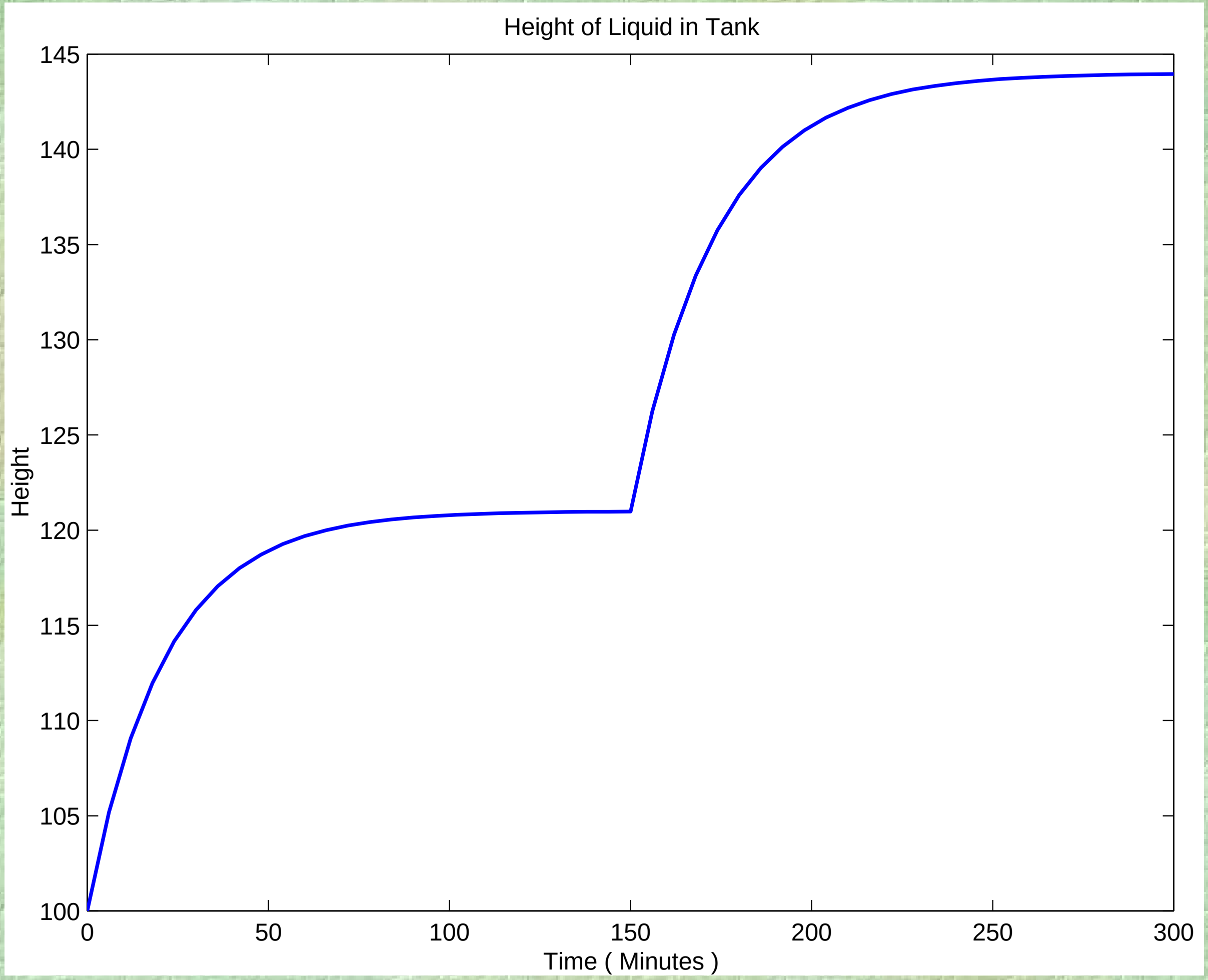
Observations

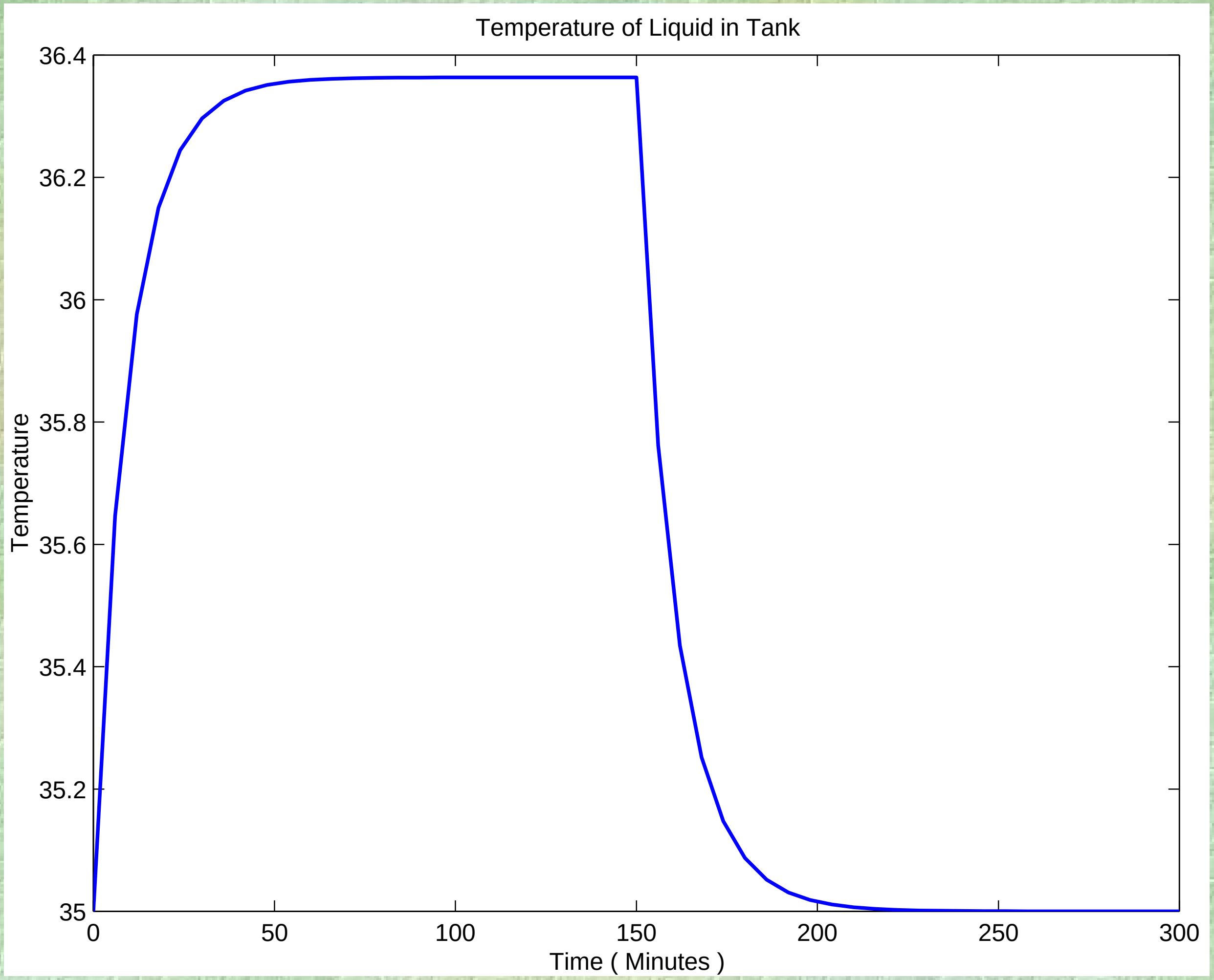
We see that the response is now much more nearly linear - This is because we are making a small change about an operating point.

Also, notice that the temperature now goes up and then down due to the fact that we now also have a cold flow.

Simulation 3

We next set both flows initially to 5. We then step the hot flow up to 6 followed by an increase in the cold flow to 6.

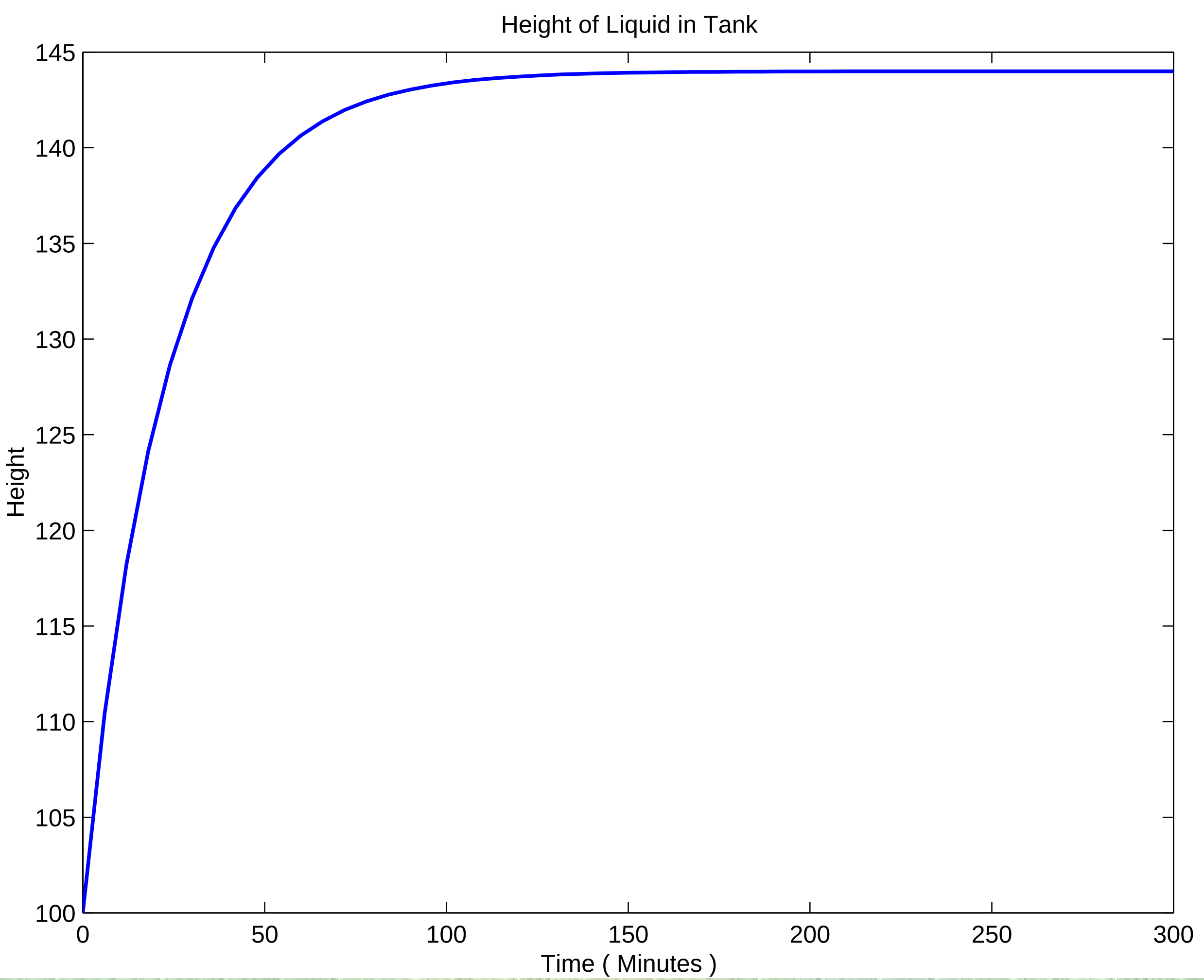

Height of Liquid in Tank
Height
Time (Minutes)
145
140
135
130
125
120
115
110
105
100
0
50
100
150
200
250
300

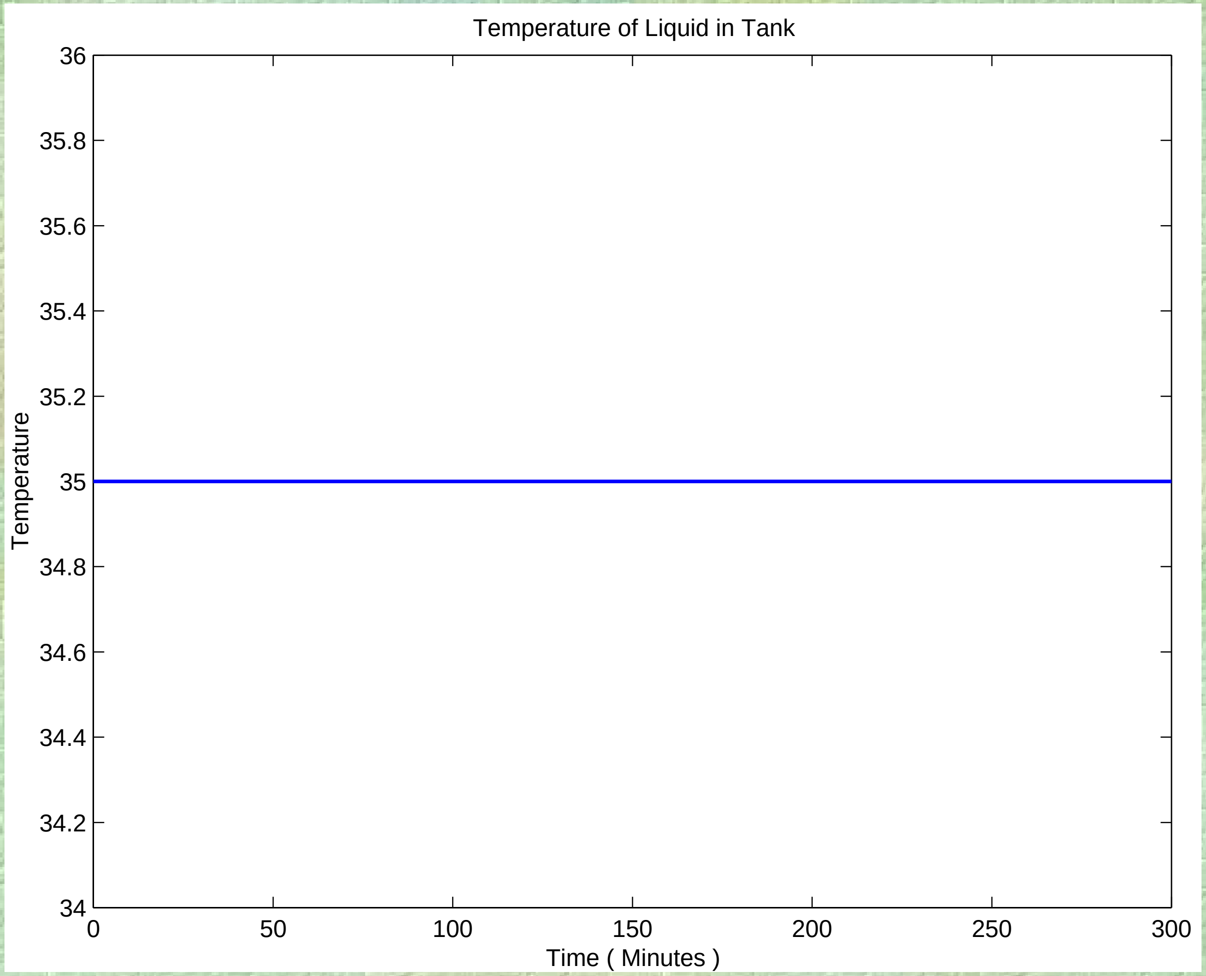

Temperature of Liquid in Tank
36.4
36.2
36
35.8
35.6
35.4
35.2
35
Temperature
0
50
100
150
200
250
300
Time (Minutes)

Simulation 4

We next step the two flows up simultaneously by the same amount.

This causes height to increase but temperature to remain constant.

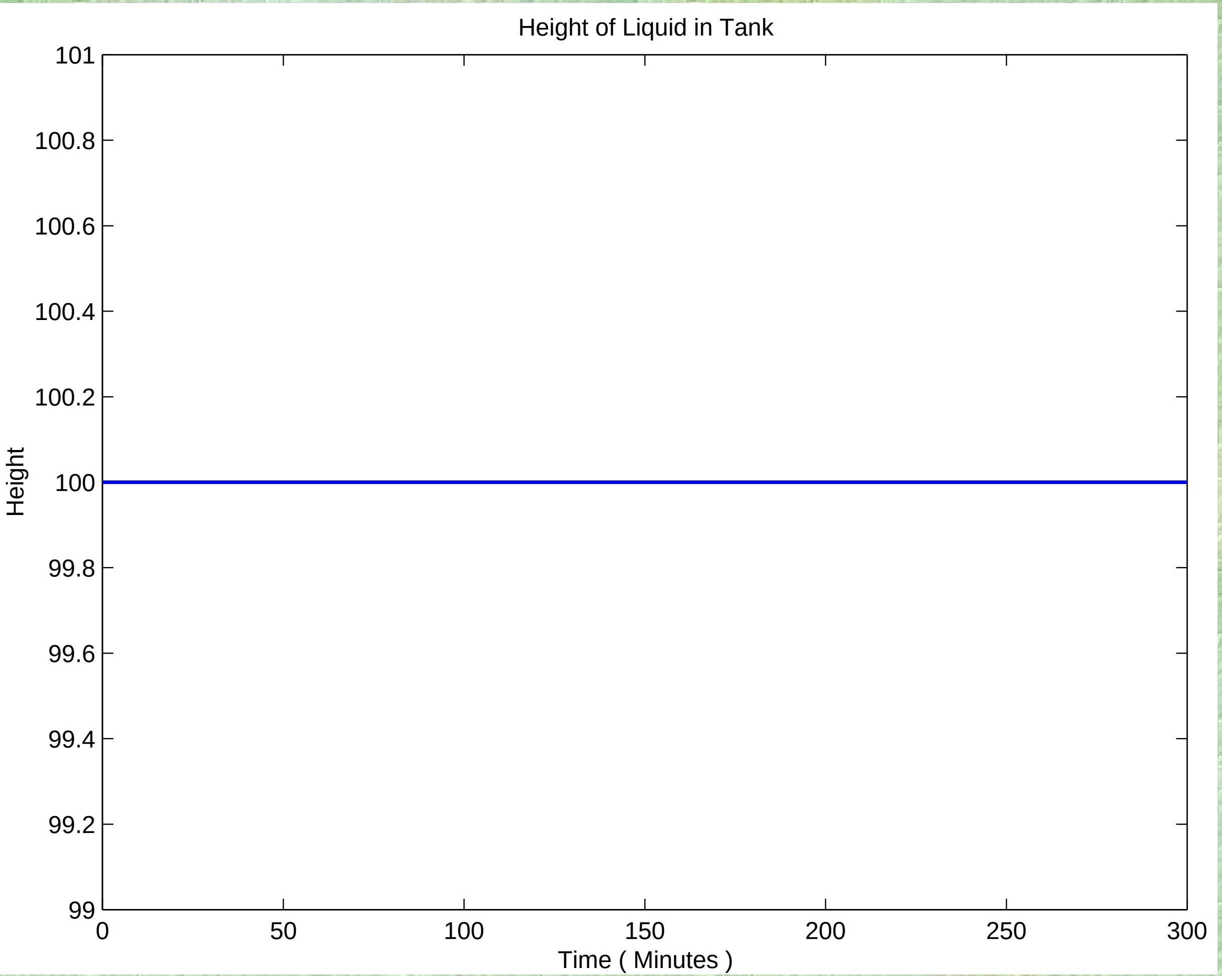

Height of Liquid in Tank
145
140
135
130
125
120
115
110
105
100
Height
0
50
100
150
200
250
300
Time (Minutes)

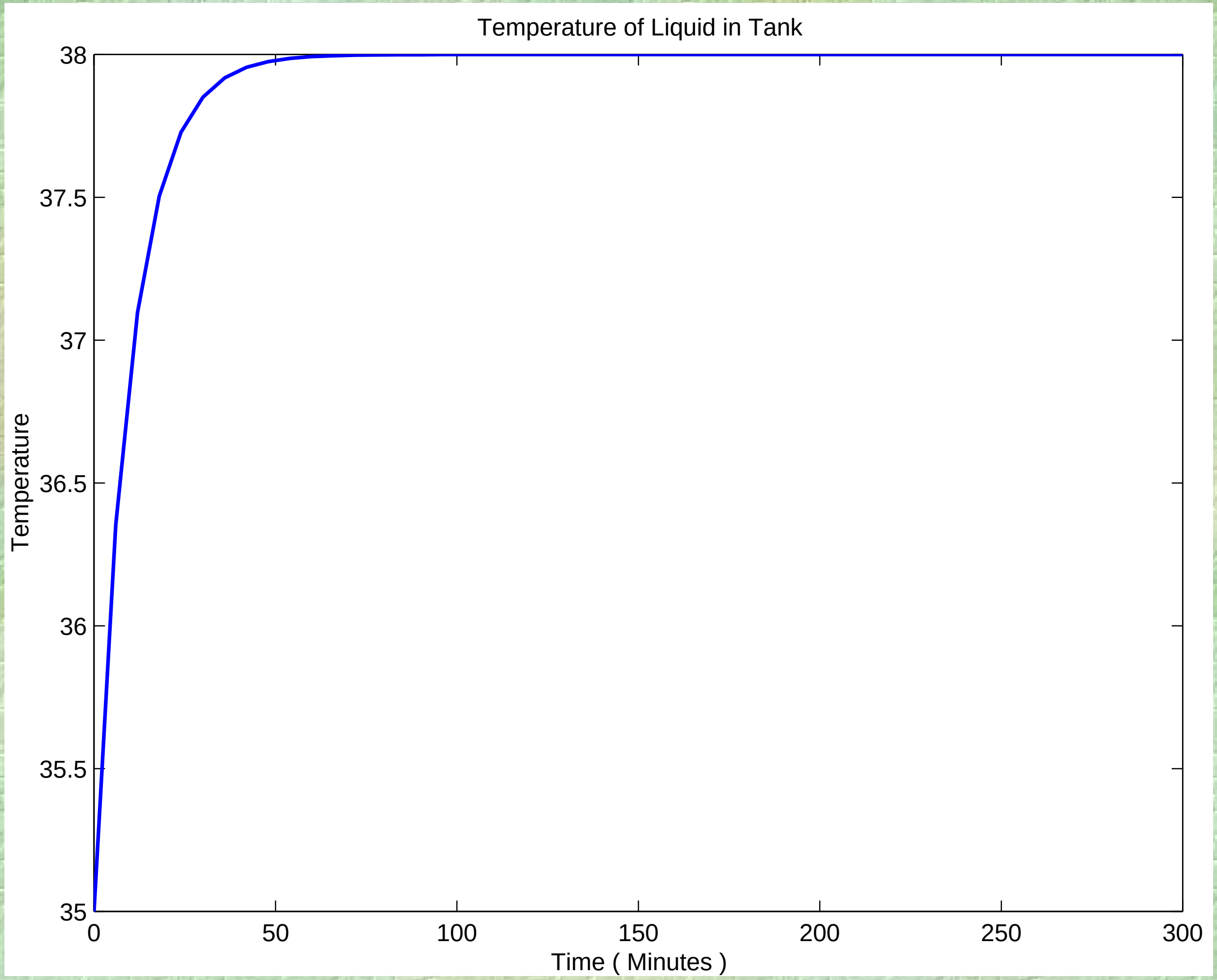

Temperature of Liquid in Tank
Temperature
Time (Minutes)
36
35.8
35.6
35.4
35.2
35
34.8
34.6
34.4
34.2
34
0
50
100
150
200
250
300

Simulation 5

We next step the hot flow up and the cold flow down by the same amount.

This causes the temperature to increase but the height to stay the same.


Height of Liquid in Tank
Height
101
100.8
100.6
100.4
100.2
100
99.8
99.6
99.4
99.2
99
0
50
100
150
200
250
300
Time (Minutes)


Temperature of Liquid in Tank
Temperature
Time (Minutes)
35
35.5
36
36.5
37
37.5
38
0
50
100
150
200
250
300

Final Observation

We see from all of these small change experiments that the system behaves in a (*very nearly*) linear fashion provided we stay near a given operating point, i.e. superposition applies.

This suggests that we should be able to find a good approximation to the model by linearizing about any given equilibrium point.

Linearization

Take $A = 1$, $k = 1$

$$h = h_0 + \Delta h \qquad h_0 = 4$$

$$T = T_0 + \Delta T \qquad T_0 = 35$$

$$u_h = u_h^0 + \Delta u_h, \quad u_h^0 = 1$$

$$u_c = u_c^0 + \Delta u_c, \quad u_c^0 = 1$$

$$T_h = 50$$

$$T_c = 20$$

Linearized Model

$$\frac{d}{dt}\begin{bmatrix}\Delta h\\ \Delta T\end{bmatrix} = A\begin{bmatrix}\Delta h\\ \Delta T\end{bmatrix} + B\begin{bmatrix}\Delta u_h\\ \Delta u_c\end{bmatrix}$$

$$A = \begin{bmatrix}-\frac{1}{4} & 0\\ 0 & -2\end{bmatrix}$$

$$B = \begin{bmatrix}1 & 1\\ 15 & -15\end{bmatrix}$$

A question to think about

Say we wanted to control the 2 flows u_h and u_c so as to:

(i) *keep the height of liquid in the tank constant*

(ii) *maintain the temperature of liquid in the tank.*

Would you use u_h to control h and u_c to control T, or vice versa ?

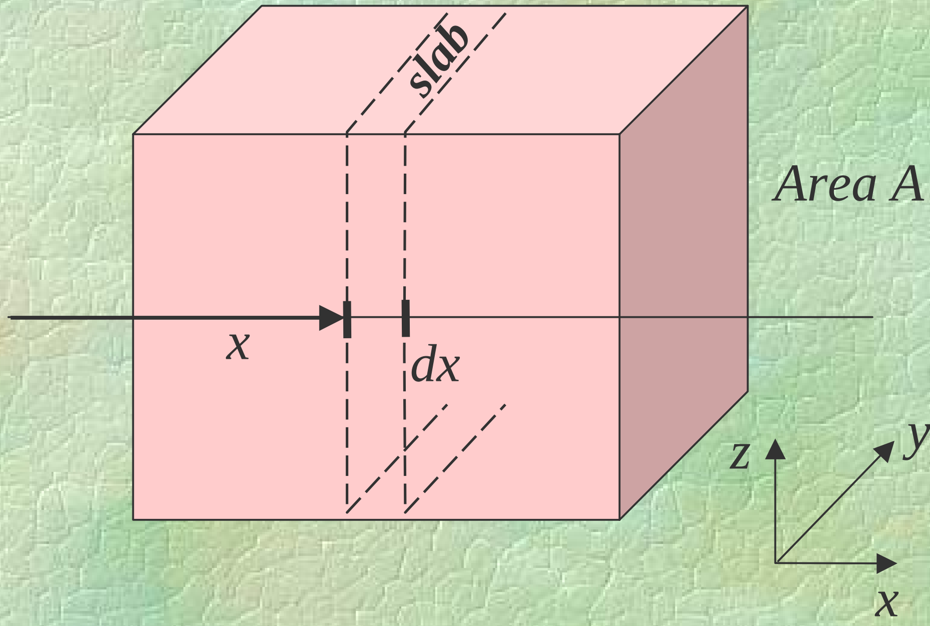
Hint

Actually this question is very interesting. It lies at the heart of architectural issues in control system design.

It may be useful to think laterally and consider what is really causing h and T to change !

Further Example 2 - One dimensional nonsteady conduction

This kind of problem occurs in many heat transfer applications. Consider a rectangular block of material of cross sectional area A where conditions ensure uniform temperatures in the y, z directions.



Let t denote time, θ temperature. Then θ is a function of x and t.

Physical Laws

Two physical laws govern this problem:

- Fourier's Law

  rate of heat conduction is proportional to temperature gradient

- First Law of Thermodynamics

  increase in internal energy

  = heat in - heat out.

A Partial Differential Equation Model

We will first derive a partial differential equation model for the temperature.

Let	dQ_x	denote heat flow into left side of slab in time dt
	dQ_{x+dx}	denote heat flow out of right side of slab in time dt.
	ρ	density of material
	c_p	specific heat

Fourier’s Law gives

$$dQ_x = -kA\left(\frac{\partial\theta}{\partial x}\right)dt$$

$$dQ_{x+dx} = -kA\frac{\partial}{\partial x}\left\{\theta + \left(\frac{\partial\theta}{\partial x}\right)dx\right\}dt$$

$$= -kA\left\{\frac{\partial\theta}{\partial x} + \left(\frac{\partial^2\theta}{\partial x^2}\right)dx\right\}dt$$

First Law of Thermodynamics gives

$$dQ_x - dQ_{x+dx} = \rho c_p (Adx)(d\theta)$$

Thus

$$\rho c_p A dx d\theta = kA \frac{\partial^2 \theta}{\partial x^2} dx dt$$

or

$$\left(\frac{\partial \theta}{dt}\right) = \alpha \left(\frac{\partial^2 \theta}{\partial x^2}\right)$$

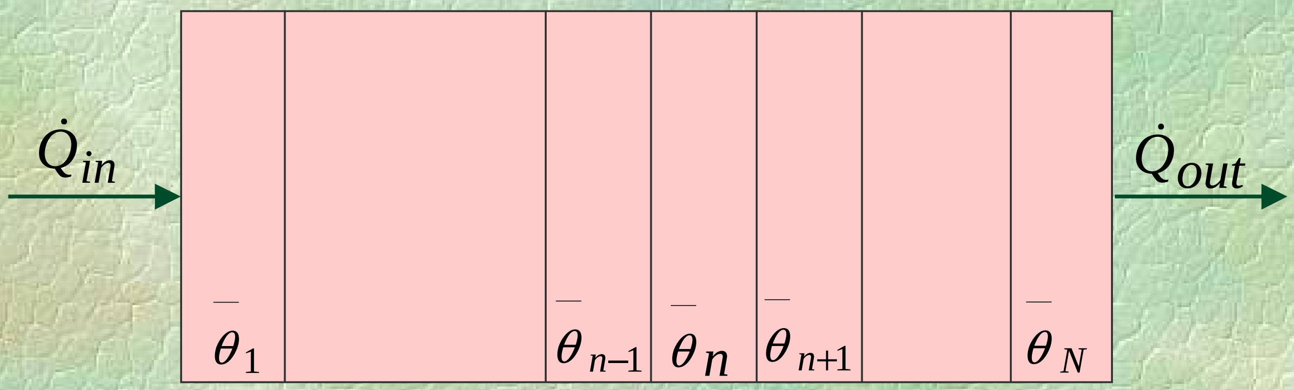
where

$$\alpha = \frac{k}{\rho c_p}.$$

Finite Difference Equation Form

Whilst there does exist some theory for the control of systems described by partial differential equations, it is usually much better to have a simplified model.

We can develop such a simplified model by dividing the slab into a finite number of strips of width Δ.



Note that θ is a function of x and t. Thus, we associate a different temperature with each slab of width Δ.

$$N\Delta = \textit{length of block}$$

We then have at time t and for the n^{th} position:

$$\frac{\partial^2 \theta}{\partial x^2} \approx \frac{\overline{\theta}_{n+1} - 2\overline{\theta}_n + \overline{\theta}_{n-1}}{\Delta^2}$$

Hence we can approximate the partial differential equation by

$$\frac{d\overline{\theta}_i}{dt} = \alpha \left[\frac{\overline{\theta}_{i+1} - 2\overline{\theta}_i + \overline{\theta}_{i-1}}{\Delta^2} \right]$$

$$i = 2, \ldots, N-1$$

At the boundaries need to include heat in and heat out, i.e.

$$\dot{Q}_{in} + kA\left[\frac{\bar{\theta}_2 - \bar{\theta}_1}{\Delta}\right] = pc_p A\Delta \frac{d\bar{\theta}_1}{dt}$$

$$-\dot{Q}_{out} + kA\left[\frac{\bar{\theta}_{N-1} - \bar{\theta}_N}{\Delta}\right] = pc_p A\Delta \frac{d\bar{\theta}_N}{dt}$$

Note also that $\dot{Q}_{in}$ and $\dot{Q}_{out}$ would typically be a function of the surface temperature plus some driving term. We then have a set of N first order linear ordinary differential equations.

When would we use this kind of model?

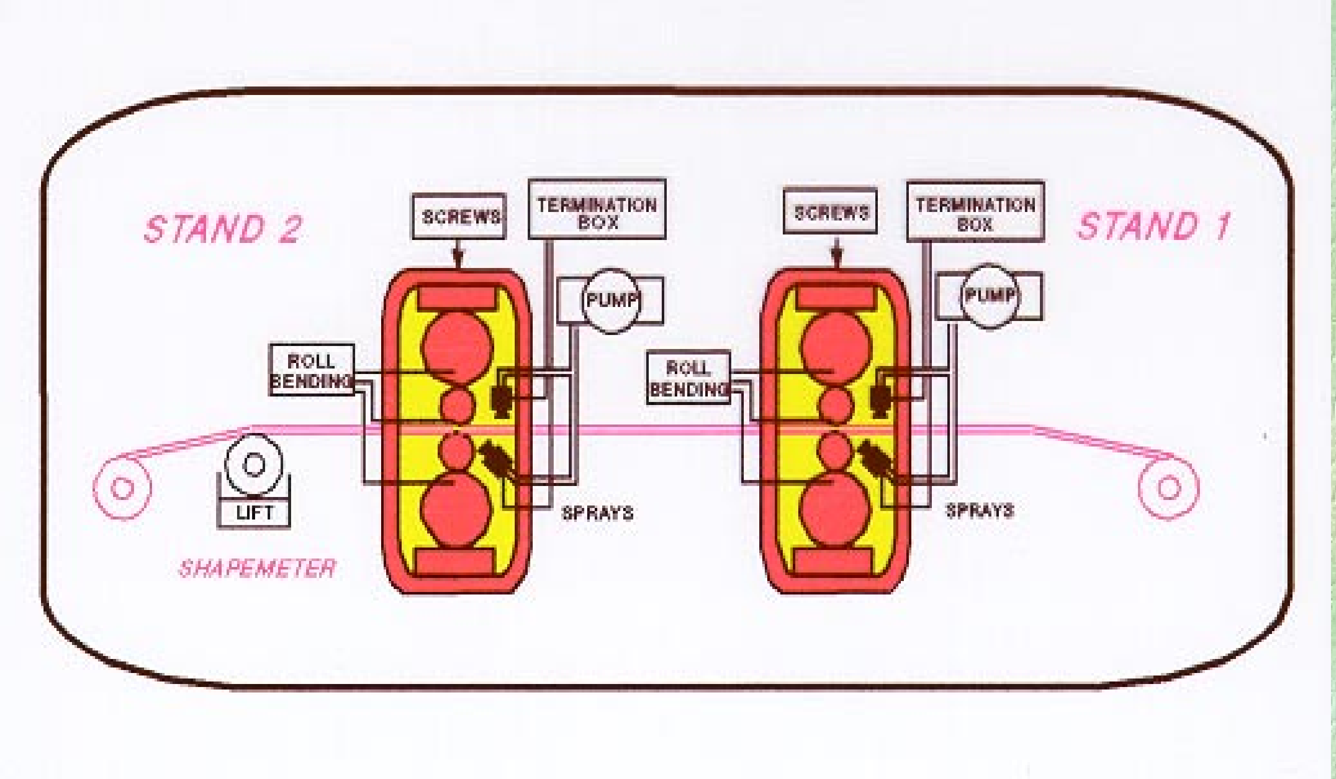
The model described above gives detailed descriptions of the time evolution of internal temperatures in the block. Thus it is probably most useful as a *calibration model*. However, for control system design one would probably use a *design model* which was much simpler. Such a model might be formed by either simulating the system and fitting a simple model or by observing the response of the real system. Typically such a model would take the form of a simple delay plus first order lag.

A real application ?

Actually the above model is essentially identical to that used to model the effect of cooling water sprays on strip shape in cold rolling mills.

[*One of the main differences is that the shape control problem actually involves cylindrical rolls rather than a rectangular slab. The reader may care to rederive the model using cylindrical coordinates*]

Typical rolling stand configuration



What is flatness in a Rolling Mill?

If rolling results in a nonuniform reduction of the strip thickness across the strip width, then a residual stress will be created, and buckling of the final product may occur. A practical difficulty is that flatness defects can be *pulled out* by the applied strip tensions, so that they are not visible to the mill operator. However, the buckling will become apparent as the coil is unwound or after it is slit or cut to length in subsequent processing operations.

Source of Flatness Problems

There are several sources of flatness problems, including the following:

- roll thermal cambers
- incoming feed disturbances (*profile*, *hardness*, *thickness*)
- transverse temperature gradients
- roll stack deflections
- incorrect ground roll cambers
- roll wear
- inappropriate mill setup (*reduction*, *tension*, *force*, *roll bending*)
- lubrication effects.

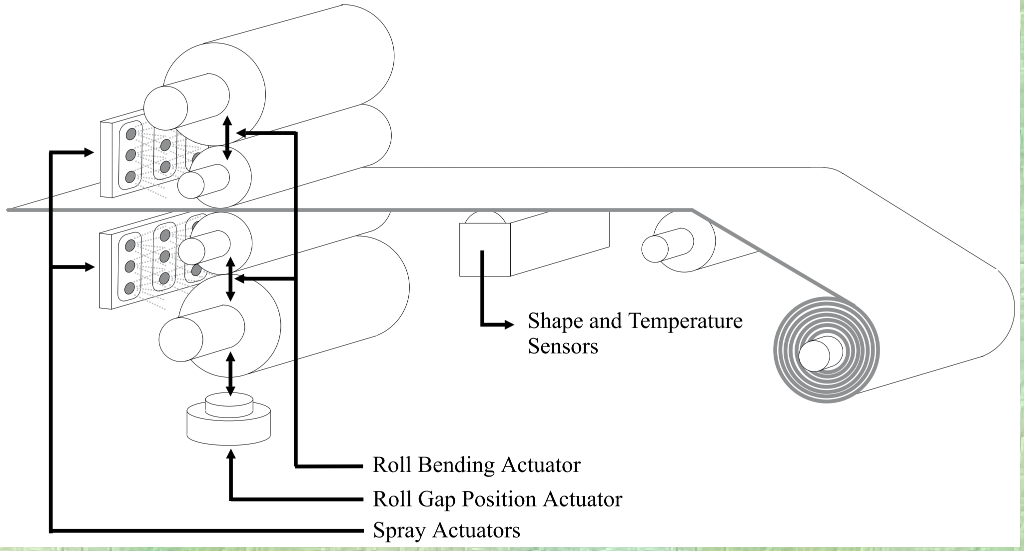
On the other hand, there are strong economic motives to control strip flatness, including the following:

- improved yield of prime-quality strip
- increased throughput, due to faster permissible acceleration, reduced threading delay, and higher rolling speed on shape-critical products
- more efficient recovery and operation on such downstream units as annealing and continuous-process lines
- reduced reprocessing of material on tension-leveling

Control Options

There are several control options to achieve improved flatness. These include roll tilt, roll bending, and cooling sprays. These typically can be separated by preprocessing the measured shape. Here, we will focus on a particular aspect of the cooling spray option. Note that flatness defects can be measured across the strip by using a special instrument called a Shape Meter. A typical control configuration is shown on the next slide.

Typical flatness-control set-up for rolling mill

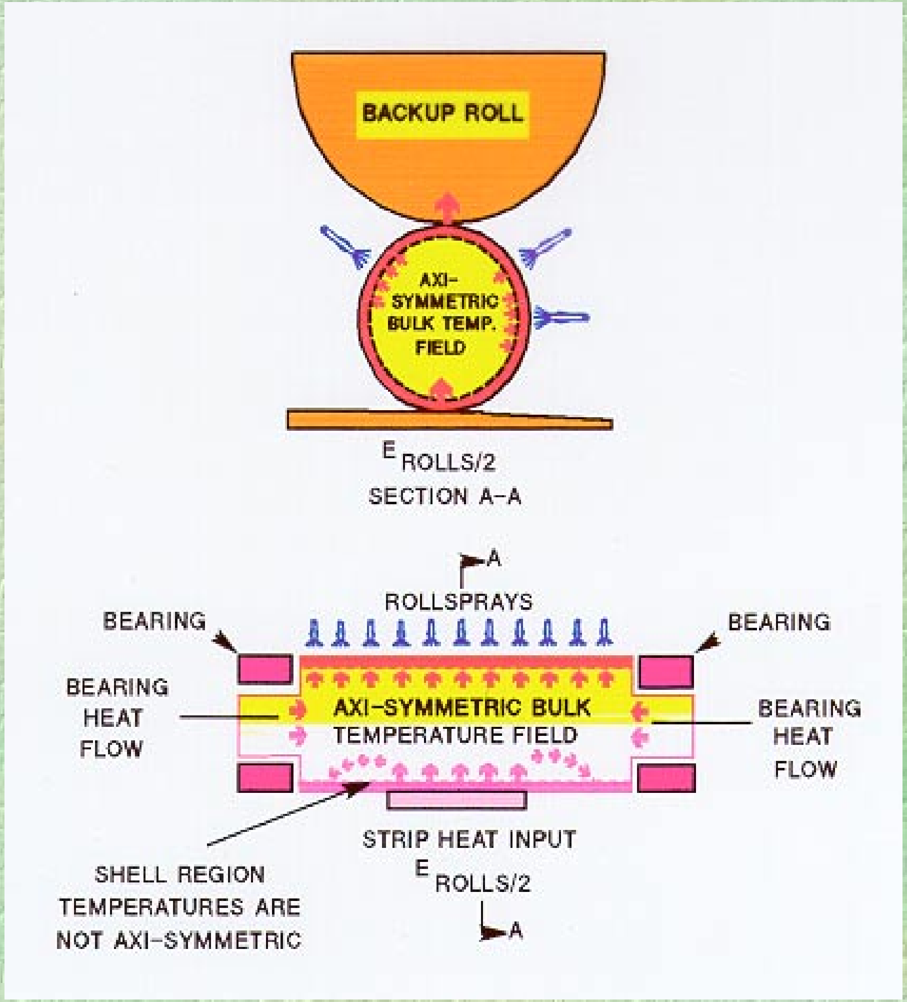


In this configuration, numerous cooling sprays are located across the roll, and the flow through each spray is controlled by a valve. The *cool* water sprayed onto the roll reduces the thermal expansion. The interesting thing is that each spray affects a large section of the roll, not just the section directly beneath it. This leads to an interactive MIMO system, rather than a series of decoupled SISO systems.

The thermal properties of the roll can be modeled using basic laws of physics. This leads to a partial differential equation, however, this can be discretized to give a finite dimensional model. Such a model can then be used as a *calibration model* to test control system design strategies.

The main components of the heat flow inside a typical roll are shown on the next slide.

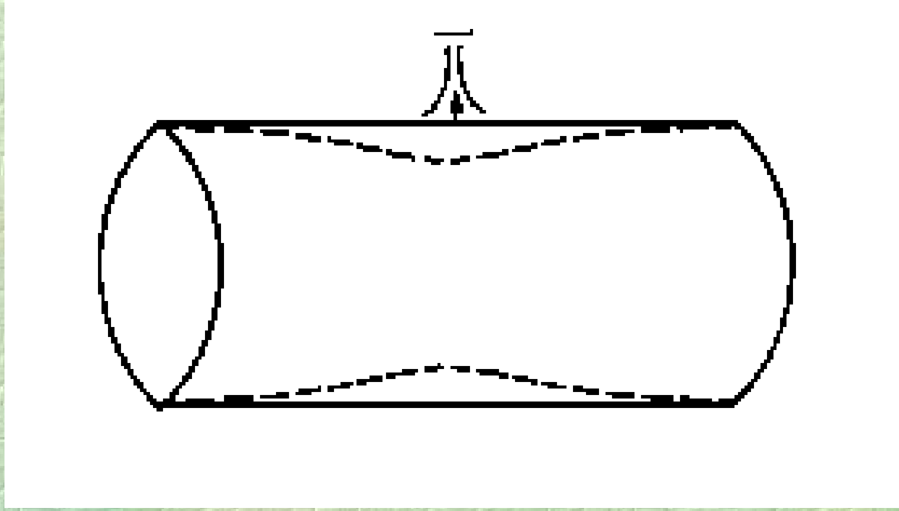
Internal roll heat flows



For the purpose of control system design, it suffices to use a simpler model. Such a model can be developed by approximating the observed behavior of the more complex *calibration model*. A key feature of the observed behavior is that a single cooling spray (*one of the actuators*) effects the radial diameter of the roll and hence the measured strip shape over a extended spatial area. This is diagrammatically shown on the next slide.

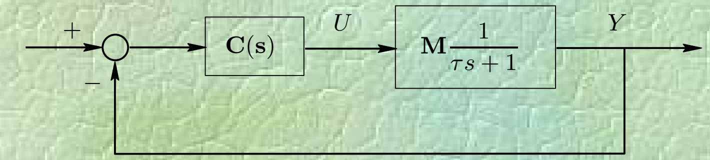
Effect of a single spray on roll diameter

Action of single spray



Based on the above discussion, a simplified model for this system (*ignoring nonlinear heat-transfer effects*, *etc*.) is shown in the block diagram on the next slide, where U denotes a vector of spray valve positions and Y denotes the roll-thickness vector. (*The lines indicate vectors rather than single signals*).

Simplified flatness-control feedback loop



The sprays affect the roll in a roughly exponential fashion as described by the matrix **M**:

$$\mathbf{M} = \begin{bmatrix} 1 & \alpha & \alpha^2 & \cdots \\ \alpha & 1 & & \\ \alpha^2 & & \ddots & \vdots \\ \vdots & & 1 & \alpha \\ & \cdots & \alpha & 1 \end{bmatrix}$$

The parameter α represents the level of interactivity in the system and is determined by the number of sprays present and how close together they are.

Control System Design

Details of the control system design for this problem are given in Chapter 21 of these notes.

Summary

- In order to systematically design a controller for a particular system, one needs a formal - though possibly simple - description of the system. Such a description is called a model.
- A model is a set of mathematical equations that are intended to capture the effect of certain system variables on certain other system variables.

- The italicized expressions above should be understood as follows:
  - *Certain system variables*: It is usually neither possible nor necessary to model the effect of every variable on every other variable; one therefore limits oneself to certain subsets. Typical examples include the effect of input on output, the effect of disturbances on output, the effect of a reference signal change on the control signal, or the effect of various unmeasured internal system variables on each other.

- *Capture*: A model is never perfect and it is therefore always associated with a modeling error. The word capture highlights the existence of errors, but does not yet concern itself with the precise definition of their type and effect.
- *Intended*: This word is a reminder that one does not always succeed in finding a model with the desired accuracy and hence some iterative refinement may be needed.
- *Set of mathematical equations*: There are numerous ways of describing the system behavior, such as linear or nonlinear differential or difference equations.

- Models are classified according to properties of the equation they are based on. Examples of classification include:

Model Attribute	*Contrasting Attribute*	*Asserts whether or not …*
Single input Single output	Multiple input multiple output	… the model equations have one input and one output only
Linear	Nonlinear	… the model equations are linear in the system variables
Time varying	Time invariant	… the model parameters are constant
Continuous	Sampled	… model equations describe the behavior at every instant of time, or only in discrete *samples* of time
Input-output	State space	… the model equations rely on functions of input and output variables only, or also include the so called *state variables*.
Lumped parameter	Distributed parameter	… the model equations are ordinary or partial differential equations

- In many situations nonlinear models can be linearized around a user defined operating point.